

For a certain popular class, the registration window is only open for 5 days (to prevent excessive enrollment).

SCORE: ____ / 30 PTS

Suppose a student is randomly selected among those who enrolled during that time. Let X be the time (measured in days) after registration opened that the student enrolled for the class. Suppose that the probability density function for X is given by

$$f(x) = \begin{cases} \frac{1}{(1+kx)^2}, & x \in [0, 5] \\ 0, & x \notin [0, 5] \end{cases}$$

- [a] Find the probability that a student enrolled for the class during the last 24 hours that registration was open.

$$\int_0^5 \frac{1}{(1+kx)^2} dx = 1 \quad (4)$$

$$\int_4^5 \frac{1}{(1+\frac{4}{5}x)^2} dx = -\frac{5}{4} (1+\frac{4}{5}x)^{-1} \Big|_4^5$$

$$= -\frac{5}{4} \left(\frac{1}{5} - \frac{5}{21} \right)$$

$$= -\frac{5}{4} \cdot \frac{-4}{105} = \frac{1}{21} \quad (3)$$

$$\textcircled{3} \quad -\frac{1}{k} \frac{1}{1+kx} \Big|_0^5 = 1$$

$$-\frac{1}{k} \left(\frac{1}{1+5k} - 1 \right) = 1$$

$$-\frac{1}{k} \cdot -\frac{5k}{1+5k} = 1$$

$$5 = 1 + 5k$$

$$k = \frac{4}{5} \quad (3)$$

- [b] Find the median time that registration was open before a student enrolled in the class.

$$\int_0^m \frac{1}{(1+\frac{4}{5}x)^2} dx = \frac{1}{2} \quad (4)$$

$$-\frac{5}{4} (1+\frac{4}{5}x)^{-1} \Big|_0^m = \frac{1}{2}$$

$$\frac{1}{1+\frac{4}{5}m} - 1 = -\frac{2}{5} \quad (3)$$

$$\frac{1}{1+\frac{4}{5}m} = \frac{3}{5}$$

$$1 + \frac{4}{5}m = \frac{5}{3}$$

$$\frac{4}{5}m = \frac{2}{3}$$

$$m = \frac{5}{6} \text{ DAYS} \quad (1)$$

$$\textcircled{3}$$

Find the length of the curve $x = 3 - 12e^t$
 $y = 9t - 2e^{2t}$ for $t \in [0, 2]$.

SCORE: ____ / 20 PTS

Your final answer must be a number, not an integral.

$$\begin{aligned} & \int_0^2 \sqrt{(-12e^t)^2 + (9 - 4e^{2t})^2} dt \\ &= \int_0^2 \sqrt{144e^{2t} + 81 - 72e^t + 16e^{2t}} dt \\ &= \int_0^2 \sqrt{81 + 72e^t + 16e^{2t}} dt \\ &= \int_0^2 (9 + 4e^{2t}) dt \\ &= (9t + 2e^{2t}) \Big|_0^2 = 18 + 2e^4 - 2 = 16 + 2e^4 \end{aligned}$$

③ EACH UNLESS OTHERWISE NOTED

②

The region bounded by $y = x^2 + 1$, $y = 18x - 80$ and $y = 10$ is revolved around the line $y = 2$.

SCORE: ____ / 35 PTS

[a] Write, **BUT DO NOT EVALUATE**, a single integral for the volume of the resulting solid.

$$x^2 + 1 = 18x - 80$$

$$x^2 - 18x + 81 = 0 \quad (3)$$

$$(x - 9)^2 = 0$$

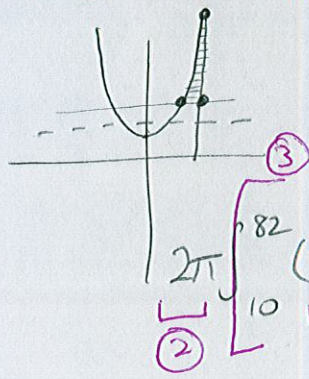
$$x = 9 \rightarrow y = 82$$

$$x^2 + 1 = 10$$

$$x = \pm 3 \rightarrow y = 10$$

$$18x - 80 = 10$$

$$x = 5 \rightarrow y = 10$$



$$y = 18x - 80 \rightarrow x = \frac{y + 80}{18}$$

$$y = x^2 + 1 \rightarrow x = \pm \sqrt{y - 1}$$

$$2\pi \int_{10}^{82} (y - 2) \left(\frac{y + 80}{18} - \sqrt{y - 1} \right) dy$$

[b]

If you used the disk or washer method in [a], write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the same volume using the shell method.

If you used the shell method in [a],

write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the same volume using the disk or washer method.

$$2\pi \int_3^5 ((x^2 + 1 - 2)^2 - (10 - 2)^2) dx + 2\pi \int_5^9 ((x^2 + 1 - 2)^2 - (18x - 80 - 2)^2) dx$$

A solid of revolution has volume $\int_2^4 \pi((\sqrt{y} + 7)^2 - ((4 - 2y) + 7)^2) dy$.

SCORE: ____ / 15 PTS

Find the equation of the axis of revolution, and the equations of the boundaries of the region being revolved.

Sketch & shade in the region being revolved.

Do NOT use the x - nor y -axes as boundaries nor the axis of revolution.

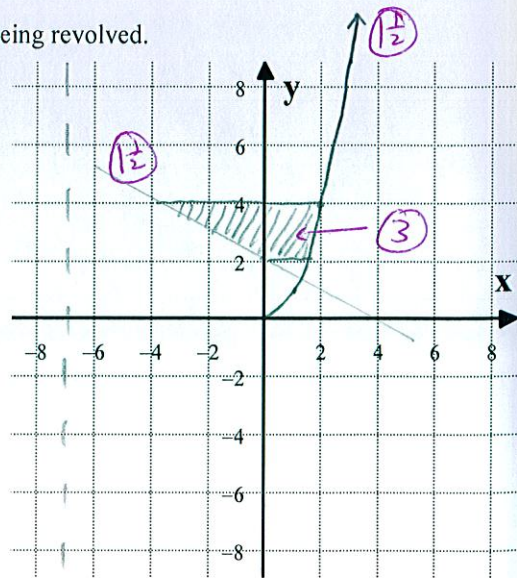
Equation of axis of revolution: ③ $x = -7$

Equations of boundaries: ② $x = \sqrt{y} \rightarrow y = x^2, x \geq 0$

② $x = 4 - 2y \rightarrow y = 2 - \frac{1}{2}x$

① $y = 2$

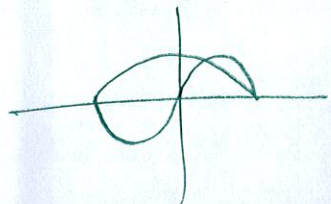
① $y = 4$



Find the area of the region between $y = \sin 2x$ and $y = \cos x$ for $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

SCORE: ____ / 25 PTS

Your final answer must be a number, not an integral. HINT: The answer is NOT 2.



$$\sin 2x = \cos x$$

$$2 \sin x \cos x = \cos x$$

$$2 \sin x \cos x - \cos x = 0$$

$$\cos x (2 \sin x - 1) = 0$$

$$\cos x = 0 \text{ or } \sin x = \frac{1}{2}$$

$$x = \pm \frac{\pi}{2}$$

$$x = \frac{\pi}{6}$$

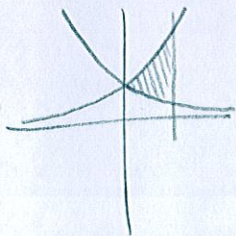
$$\begin{aligned} & \int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} (\cos x - \sin 2x) dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin 2x - \cos x) dx \\ &= \left(\sin x + \frac{1}{2} \cos 2x \right) \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{6}} + \left(-\frac{1}{2} \cos 2x - \sin x \right) \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} \\ &= \left(\frac{1}{2} + \frac{1}{4} - (-1 - \frac{1}{2}) \right) + \left(\frac{1}{2} - 1 - (-\frac{1}{4} - \frac{1}{2}) \right) \\ &= \frac{9}{4} + \frac{1}{4} \\ &= \frac{5}{2} \quad \textcircled{4} \end{aligned}$$

③ EACH UNLESS OTHERWISE NOTED

Find the y-coordinate of the center of mass of the region defined by $y \leq e^x$, $y \geq e^{-x}$ and $x \leq 1$.

SCORE: ____ / 25 PTS

Your final answer must be a number, not an integral.



$$\int_0^1 (e^x - e^{-x}) dx$$

$$= (e^x + e^{-x}) \Big|_0^1$$

$$= e + \frac{1}{e} - (1 + 1)$$

$$= \frac{e^2 - 2e + 1}{e}$$

$$= \frac{(e-1)^2}{e}$$

$$\bar{y} = \frac{1}{4} \frac{(e^2-1)^2}{e^2} \cdot \frac{e}{(e-1)^2}$$

$$= \frac{(e+1)^2}{4e}$$

④

$$\frac{1}{2} \int_0^1 (e^{2x} - e^{-2x}) dx$$

$$= \frac{1}{2} \left(\frac{1}{2} e^{2x} + \frac{1}{2} e^{-2x} \right) \Big|_0^1$$

$$= \frac{1}{4} \left(e^2 + \frac{1}{e^2} - (1 + 1) \right)$$

$$= \frac{1}{4} \left(\frac{e^4 - 2e^2 + 1}{e^2} \right)$$

$$= \frac{1}{4} \frac{(e^2-1)^2}{e^2}$$

③ 1/2 EACH

UNLESS

OTHERWISE

NOTED